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PULSE EXPANSION OF A THERMOELASTIC VISCOPLASTIC CYLINDRICAL SHELL BY INTERNAL PRESSURE

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Abstract

A conjugate problem of predicting the kinematic and thermal parameters as well as the parameters of the stressed-strained state of a cylindrical shell made of thermoelastic viscoplastic material and loaded by an internal pressure pulse has been solved in terms of waves.

1. Introduction

The subject of this research is a thin-wall cylindrical shell made of elastic viscoplastic material of the Kristesku-Malvern-Sokolovski type [1], with the latter's indicial equation describing the properties typical for most metals – strain and dynamic hardening, as well as thermal weakening in the process of loading, with the latter being caused by dissipation of inelastic deformation energy into thermal energy, having taken into account the heat transfer attending the process. The outer and inner radii of the shell have been designated R_0 and r_0, accordingly. At a time moment $t = 0$, a radial pressure pulse $p = p(t)$, with $p(0) \neq 0$, has been applied to the shell's internal surface. The pulse's amplitude parameters are such, so as to take the shell out of the elastic state.

2. Problem positing

The solution has been carried out in a cylindrical coordinate system *(R, θ, z)* within the framework of the hypothesis of planar stressed state. The stress tensor's components are σ_{RR} and $\sigma_{\theta\theta}$ which are different from zero (hereinafter referred to as σ_R and σ_θ, accordingly), and the strain tensor's components are ε_{RR}, $\varepsilon_{\theta\theta}$ and ε_{ZZ} (ε_R, ε_θ, and ε_Z, accordingly).

The equation of motion of a circular element of the shell with radius $R \in [r_0; R_0]$ and thickness *dR* has been expressed as follows:

$$\rho \frac{\partial V_R}{\partial t} = \frac{\partial \sigma_R}{\partial R} + \frac{\sigma_R - \sigma_\theta}{R} \tag{1}$$

where V_R –radial velosity; ρ - material density

The relative strains ε_R and ε_θ and velocity V_R are interrelated by conditions for continuity:

$$\frac{\partial \varepsilon_R}{\partial t} = \frac{\partial V_R}{\partial R}; \frac{\partial \varepsilon_\theta}{\partial t} = \frac{V_R}{R} \tag{2}$$

The constitutive equations for the shell material have been assumed in the form [2]:

$$\frac{\partial \varepsilon_R}{\partial t} = \frac{1}{E\left(\varepsilon_i^p, T\right)}\left(\frac{\partial \sigma_R}{\partial t} - \mu \frac{\partial \sigma_\theta}{\partial t}\right) + \left[\frac{\sqrt{J_2} - \sigma\left(\varepsilon_i\right)}{\beta\left(\varepsilon_i\right)}\right]^{\alpha(\varepsilon_i)} *$$

$$* \frac{2\sigma_R - \sigma_\theta}{6\sqrt{J_2}}\left[1 - \frac{1}{E^2\left(\varepsilon_i^p, T\right)} \frac{\partial E\left(\varepsilon_i^p, T\right)}{\partial \varepsilon_i^p}\left(\sigma_R - \mu \sigma_\theta\right)\right];$$

$$\frac{\partial \varepsilon_\theta}{\partial t} = \frac{1}{E\left(\varepsilon_i^p, T\right)}\left(\frac{\partial \sigma_\theta}{\partial t} - \mu \frac{\partial \sigma_R}{\partial t}\right) + \left[\frac{\sqrt{J_2} - \sigma\left(\varepsilon_i\right)}{\beta\left(\varepsilon_i\right)}\right]^{\alpha\left(\varepsilon_i\right)} *$$

(3) $$* \frac{2\sigma_\theta - \sigma_R}{6\sqrt{J_2}}\left[1 - \frac{1}{E^2\left(\varepsilon_i^p, T\right)} \frac{\partial E\left(\varepsilon_i^p, T\right)}{\partial \varepsilon_i^p}\left(\sigma_\theta - \mu \sigma_R\right)\right];$$

$$\frac{\partial \varepsilon_z}{\partial t} = -\frac{\mu}{E\left(\varepsilon_i^p, T\right)}\left(\sigma_R + \sigma_\theta\right) - \left[\frac{\sqrt{J_2} - \sigma\left(\varepsilon_i\right)}{\beta\left(\varepsilon_i\right)}\right]^{\alpha\left(\varepsilon_i\right)} *$$

$$* \frac{\sigma_R + \sigma_\theta}{6\sqrt{J_2}}\left[1 - \frac{1}{E^2\left(\varepsilon_i^p, T\right)} \frac{\partial E\left(\varepsilon_i^p, T\right)}{\partial \varepsilon_i^p}\left(\sigma_\theta + \sigma_R\right)\right],$$

where I_2 – invariant of the stress deviator tensor;

ε_i – strain intensity;

ε^p_i – reduced plastic deformation;

μ – Poisson's ratio;

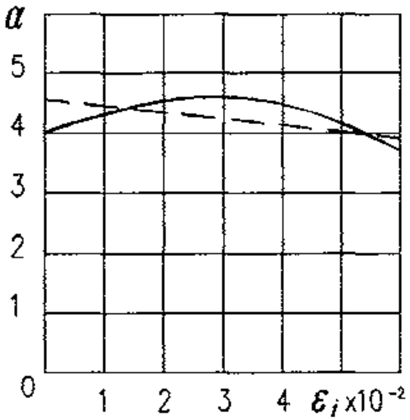


Fig. 1. Approximation of relation $\alpha = \alpha(\varepsilon)$ *for steel 3*

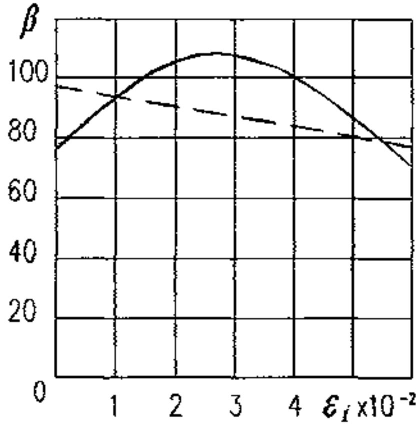


Fig. 2. Approximation of relation $\beta = \beta(\varepsilon)$ *for steel 3*

The approximating functions $\alpha = \alpha(\varepsilon)$, $\beta = \beta(\varepsilon)$ and $\sigma = \sigma(\varepsilon)$ have been expressed with reference to steel 3 in the form [2]:

(4) $$\alpha = 4{,}5 - 7\varepsilon_i\ ;\ \ \beta = 98{,}1 - 232\varepsilon_i;\ \ \sigma = 241 + 2196\varepsilon_i,\ \text{[MPa]}.$$

Figures 1…, 3 illustrate the satisfactory matching between the results from the experiment (solid lines) and theoretical (design) relations (4) (dash lines). The relation between the modulus of elasticity and plastic deformation value has been expressed by means of the exponential function in the form [3]:

The boundary conditions of the problem have been assumed in the following form

(5) $$\sigma_R(r_0,t)=\sigma_{Ro}H(t);\sigma_R(R_0,t)=0,$$

and the initial conditions –

(6) $$\sigma_R(r_0,0)=\sigma_\theta(r_0,0)=V_R(r_0,0)=\varepsilon_R(r_0,0)=\varepsilon_\theta(r_0,0)=\varepsilon_z(r_0,0)=0$$

where *H(t)* – Heaviside single function.

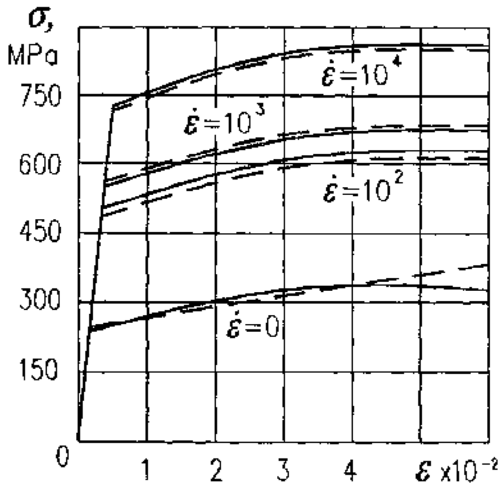


Fig. 3. Approximation of dynamic load diagrams for steel 3 by means of constitutive equations (3), (4)

The system of equations (1),...,(3) is quasi-linear of the hyperbolic type. It has been solved numerically by the characteristics method [4] with the subsequent finite-difference approximation of the differential relations between the sought functions along the characteristic directions.

The latter are in the following form:

- along the divisible characteristics $dR = 0$:

$$d\varepsilon_\theta = \frac{V_R}{R}dt;$$

$$d\varepsilon_{\theta} = \frac{d\sigma_{\theta} - \mu d\sigma_{R}}{E} + \left[\frac{\sqrt{J_2} - \sigma(\varepsilon_i)}{\beta(\varepsilon_i)} \right]^{\alpha(\varepsilon_i)} *$$

$$* \frac{2\sigma_{\theta} - \sigma_{R}}{6\sqrt{J_2}} \left[1 - \frac{1}{E^2} \frac{\partial E}{\partial \varepsilon_i^p} (\sigma_{\theta} - \mu\sigma_{R}) \right] dt;$$

$$d\varepsilon_{R} = \frac{d\sigma_{R} - \mu d\sigma_{\theta}}{E} + \left[\frac{\sqrt{J_2} - \sigma(\varepsilon_i)}{\beta(\varepsilon_i)} \right]^{\alpha(\varepsilon_i)} *$$

$$* \frac{2\sigma_{R} - \sigma_{\theta R}}{6\sqrt{J_2}} \left[1 - \frac{1}{E^2} \frac{\partial E}{\partial \varepsilon_i^p} (\sigma_{R} - \mu\sigma_{\theta}) \right] dt;$$

$$d\varepsilon_{z} = -\mu \frac{d\sigma_{\theta} + \mu d\sigma_{R}}{E} - \left[\frac{\sqrt{J_2} - \sigma(\varepsilon_i)}{\beta(\varepsilon_i)} \right]^{\alpha(\varepsilon_i)} *$$

(7) $$* \frac{2\sigma_{R} + \sigma_{\theta}}{6\sqrt{J_2}} \left[1 + \frac{1}{E^2} \frac{\partial E}{\partial \varepsilon_i^p} (\sigma_{R} - \sigma_{\theta}) \right] dt;$$

- along the characteristics $dR = \pm\sqrt{\frac{E}{\rho(1-\mu^2)}} dt$:

$$\mp dV_{R} + \sqrt{\frac{1-\mu^2}{E\rho}} d\sigma_{R} + \frac{\sigma_{R} - \sigma_{\theta}}{R\rho} dt \mp \sqrt{\frac{E}{\rho(1-\mu^2)}} \mu \frac{V_{R}}{R} dt +$$

$$(8) \qquad +\sqrt{\frac{E}{\rho(1-\mu^2)}}\left[\frac{\sqrt{J_2}-\sigma(\varepsilon_i)}{\beta(\varepsilon_i)}\right]^{\alpha(\varepsilon_i)}(A+B)dt=0.$$

where

$$A=\frac{2\sigma_R-\sigma_\theta}{6\sqrt{J_2}}\left[1-\frac{1}{E^2}\frac{\partial E}{\partial\varepsilon_i^p}\left(\sigma_R-\mu\sigma_\theta\right)\right];$$

$$B=\mu\frac{2\sigma_\theta-\sigma_R}{6J_2}\left[1-\frac{1}{E^2}\frac{\partial E}{\partial\varepsilon_i^p}\left(\sigma_\theta-\mu\sigma_R\right)\right]$$

Integration of the system of six ordinary differential equations (7), (8) along the curvilinear characteristic directions has been carried out numerically. Moreover, at each integration step, a cycle of iterations to ε^p_i has been organized and temperature T has risen as a result of the dissipation of inelastic deformation energy into thermal energy. Furthermore, heat redistribution in the shell due to heat transfer has been taken into consideration by means of the heat flow equation in the form [5]:

$$(9) \qquad \frac{\partial T}{\partial t}+\frac{a}{\omega^2}\frac{\partial^2 T}{\partial t^2}=a\left(\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}\right)$$

in which the term $\frac{a}{\omega^2}\frac{\partial^2 T}{\partial t^2}$ takes into account the finiteness of the rate of heat transfer; a – coefficient of temperature conductivity. In calculations, the rate of heat flow has been assumed to be ω= 1800 m/s (divisible by the velocity of propagation of radial pressure wave in the shell material which for steel 3 is equal to c = 5400 m/s).

It is convenient to solve equation (9) by the characteristics method, too. The characteristic directions and differential relations between the sought functions

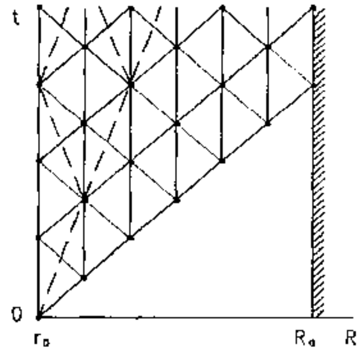
$$v = \frac{\acute{\Gamma}Ð}{\acute{\Gamma}Ð} \qquad u = \frac{\acute{\Gamma}Ð}{\acute{\Gamma}Ð}$$

along them are in the following form:

$$dR = \pm \omega dt;$$

(10) $$\pm\, du - \frac{1}{\omega} dV \pm \frac{\omega^2}{a} udt + \omega \frac{u}{R} dt = 0.$$

In Fig. 4, it is plotted in the phase plane *R, t,* the combined grid of characteristics for a conjugate solution of the system of equations (7), (8) and (10). The selection of a numerical integration step for equations (10) with relation to the integration step of equations (7), (8) is secondary, assuming that the characteristics lines (10) pass through the node points of the characteristics of equations (7), (8). At the initial iteration step in the current node point, the characteristics of the stressed-strained and kinematic state have been determined, as well as the temperature caused by inelastic deformation of the material, the temperature being driven to the given point as a result of heat transfer. The temperatures have been then summarized, and according to the summary temperature the values of the six sought functions σ_R, σ_θ, V_R, ε_R, ε_θ and ε_Z have been defined more precisely, then the values of temperature T and modulus of elasticity E have been defined more precisely – and the iteration cycle has been repeated until the required precision is achieved. The heat interaction of the shell with the environment has been assumed to be lacking.



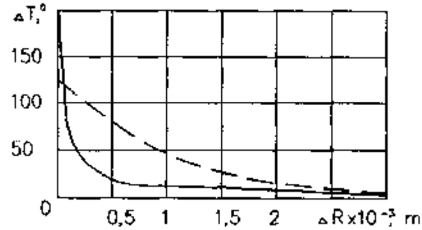
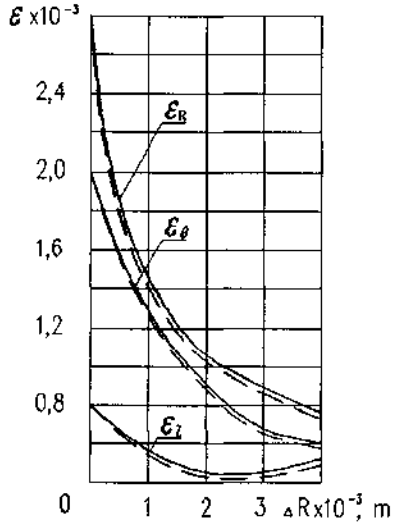


Fig. 4. Combined grid of characteristics in a phase plane of independent variables

Fig. 5. Effect of heat transfer on temperature distribution in the shell at a fixed time moment

3. Conclusion and deductions

Some results of the calculations are illustrated in Fig. 5,…, 8. In Fig. 5, snapshots of temperature distribution through the shell thickness at a fixed time moment of $t = 3.3 \times 10^{-6}$ s have been plotted. The problem has been solved for boundary conditions (5) at $\sigma_{R0} = 350$ MPa, $r_0 = 0.05$ m, $R_0 = 0.06$ m. The effect of heat transfer in the process of loading has been analyzed. The solid line in the figure has been plotted without taking into account the heat transfer ($a = 0$), and the dash one – with taking into account the heat transfer at $a = 0.024$ m^2/s. Apparently, when taking into account the heat transfer in the shell dynamic loading, it affects materially the current temperature distribution, moreover, the loaded shell surface heating decreases.



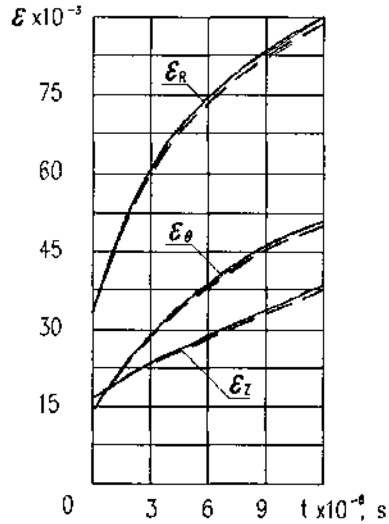


Fig. 6. Snapshots of deformation components for $t = 1.5 \times 10^{-5}$ s

Fig. 7. Variation of deformation components on boundary surface

The variation of deformations and radial stress through the shell thickness and in time is shown in Fig. 6,…,8. Snapshots showing deformation components for $t = 1.5 \times 10^{-5}\ s$ are shown in Fig. 6, the variation of deformations in time on the loaded surface $R = r_0$ is shown in Fig. 7, and the variation of radial stresses through the shell thickness at a time moment $t = 1.5 \times 10^{-5}\ s$ is shown in Fig. 8. All curves have been plotted at $a = 0$, the impact of temperature and reduced plastic deformation on the variation of the material's modulus of elasticity has been analyzed. The solid lines in the figures correspond to $E = const$, and the dash ones – to the variable modulus of elasticity. Apparently, when taking into account the shell material heating, it does not affect materially the variation of the stress-strained state parameters. Obviously, the picture will change materially in the cases, when loading of the shell's inner surface will be attended by its pulse heating by an external independent source, which is typical, for example, for the cases of gun firing or loading the shell's inner surface by products of detonation of high explosives. In these cases it should be expected (as confirmed by the carried out calculations, too) that, as a result of the intensive heating of the shell's inner surface, the inner layers of its material will be in a compressed state, including the layers along the tangential components of stress and strain, thus increasing artificially the safety factor of the shell and changing the time-sequence of appearance of microdefects and destruction through the shell space in the process of its expansion: destruction begins not from its inner surface, as it follows from the solution of the well-known Lame's problem, but in the middle layers, in the area where tangential stresses change their sign.

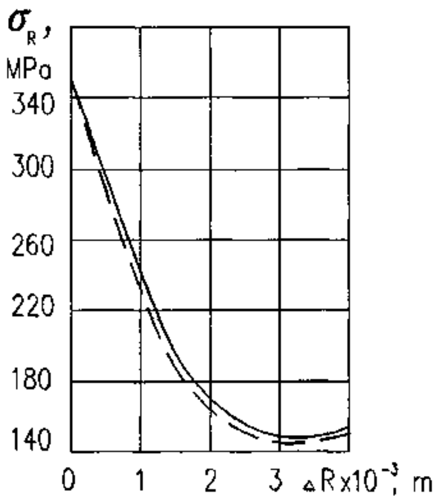


Fig. 8. Snapshots showing profiles of radial stresses waves in the shell for $t = 1.5 \times 10^{-5}$ s

Tooling has been thereby developed that allows to predict the behavior of cylindrical shells under conditions of pulse loading on the inner surface as well as to solve a series of problems of practical importance.

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ИМПУЛСНО РАЗШИРЕНИЕ НА ЦИЛИНДРИЧНА ОБЛИЦОВКА ОТ ТЕРМОЕЛАСТИЧЕН - ВЯЗКОПЛАСТИЧЕН МАТЕРИАЛ ПОД ДЕЙСТВИЕТО НА ВЪТРЕШНО НАЛЯГАНЕ

В.Баранов, И.Гецов

Резюме

Решава се вълнова задача за определяне на кинематичните параметри и параметрите на напрегнато - деформираното състояние на цилиндрицната облицовка от термоелестичен-вязкопластичен материал, натоварена от импулс на вътрешно налягане.